SURFACES OF DISCONTINUITY SEPARATING TWO PERFECT MEDIA OF DIFFERENT PROPERTIES. RECOMBINATION WAVES IN MAGNETOHYDRODYNAMICS

PMM Vol. 32, №6, 1968, pp. 1125-1131

A. G. KULIKOVSKII
(Moscow)

(Received June 27, 1968)

Magnetohydrodynamic gas-ionizing shock waves can be used as an example of a surface of discontinuity separating two perfect media of different properties [1–5]. The gasdynamic equations are, in this case, used to describe the behavior of the gas on one side of the surface of discontinuity and the magnetohydrodynamic equations, to describe it on the other side. It has been shown that in a number of cases the boundary conditions emerging from the continuity requirements are insufficient to describe the ionizing shock waves. Additional boundary conditions needed follow from the requirement of the existence of a continuous solution describing the wave structure, and their form depends on the ratios of the dissipative coefficients of the gas.

Below we consider the general properties of the surfaces of discontinuity separating two arbitrary perfect media (including the particular case of two identical media), and obtain additional relations following from the condition of existence of a solution describing the structure of such a wave. These relations depend, in general, on the dissipative properties within a narrow layer representing the shock wave.

Section 1 deals with classification of the evolutionary discontinuities separating two different media. Section 2 contains a general discussion of the structure of such discontinuities, together with additional boundary conditions, and the conditions of evolutionarity are also assessed. In Sect. 3 an example is given where recombination waves (across which the conductivity of gas varies from infinity to zero) are considered in the presence of a magnetic field of arbitrary orientation.

1. General properties and classification of the surfaces of discontinuity. Let us suppose that the surface $x = x^* (t)$ separates two perfect media, i.e. media in which plane waves of small perturbations move without attenuation or dispersion. We shall assume that n_1 variables describe the medium situated to the left of the discontinuity and n_2 variables – the medium to the right of the discontinuity and, that the number of small perturbations of various types moving in the left direction away from the discontinuity is s_1, while that moving to the right is s_2 (numbers of perturbations arriving from the left and right direction are, respectively, $n_1 - s_1$ and $n_2 - s_2$).

Usually a certain number of relations can be set up directly, for the perfect media, at the discontinuity. Relations following from the laws of conservation of mass, impulse and energy and the continuity relations for the tangential component of the electric field and normal component of the magnetic field following from the Maxwell equations, are examples of such relations. We shall call these relations fundamental and denote their number by r.

If $s_1 + s_2 + 1 < r$, then the discontinuity is nonevolutionary [6] and the amplitudes of the outgoing small perturbations are overdefined. Interaction of such discontinuities with small perturbations lead to the appearance of perturbations of finite amplitude, and the discontinuity disintegrates. Transalfvenian waves in MHD are an example of such discontinuities.

If $s_1 + s_2 + 1 > r$, then discontinuities may exist. In this case additional $r - (s_1 + s_2 + 1)$ relations are required. These relations will ensure that the discontinuity is evolutionary and will make obtaining unique solutions to problems incorporating such a discontinuity possible. The above number of relations can, indeed, be obtained from the requirement of existence of a solution representing the structure of such waves (see Sect. 2 below). These relations will depend on the character of the dissipative processes within the discontinuity. The waves for which additional relations are required include the combustion waves in gas dynamics [7] and in MHD [8–10] as well as the ionizing waves [1–5] in magnetic field. When this approach to the problem is adopted, we see that the existence of a discontinuity of one or the other type is intimately connected with the existence of solution representing the structure of the discontinuity. Existence of such structure will be discussed in very general terms in Sect. 2. Each particular case demands a more detailed investigation. Here we shall assume that the discontinuity exists and that the necessary additional relations have been obtained. If $s_1 + s_2 + 1 = r$, then r fundamental relations are sufficient for a complete description of the discontinuity.

Thus, at the evolutionary discontinuity $s_1 + s_2 + 1$ relations should hold. These relations will connect n_1 quantities to the left of the discontinuity, n_2 – to the right of the discontinuity and the velocity of the discontinuity $U = dx^*/dt$.

Let n_1 quantities to the left of the discontinuity and U, be given. If $n_2 > s_1 + s_2 + 1$, then n_2 quantities to the right of the discontinuity can be determined. This will not, however, be unique, as it will involve an arbitrariness necessary for the wave to be evolutionary and needed for obtaining solutions to problems containing such a discontinuity. A unique determination of n_2 quantities to the right of the discontinuity is possible when $n_2 = s_1 + s_2 + 1$. If however $n_2 < s_1 + s_2 + 1$, then the quantities to the right of the discontinuity cannot be defined for arbitrarily elected quantities to the left of the discontinuity and of its velocity. In order to satisfy conditions at the discontinuity it is necessary for the $n_1 + 1$ given quantities to be bound by $s_1 + s_2 + 1 - n_2$ relations. Similar conclusions can be drawn by assuming the quantities to the right of the discontinuity and its velocity to be given.

We therefore see that the possible types of discontinuities characterized by the relations below, are

$$
\begin{array}{llll}
\text{I} & n_2 = s_1 + s_2 + 1 = n_1 & & \\
\text{II} & n_2 = s_1 + s_2 + 1 < n_1 & \text{II}' & n_1 = s_1 + s_2 + 1 < n_2 \\
\text{III} & n_2 < s_1 + s_2 + 1 = n_1 & \text{III}' & n_1 < s_1 + s_2 + 1 = n_2 \\
\text{IV} & n_2 < s_1 + s_2 + 1 < n_1 & \text{IV}' & n_1 < s_1 + s_2 + 1 < n_2 \\
\text{V} & s_1 + s_2 < n_1 \quad s_1 + s_2 + 1 < n_2 & & \\
\text{VI} & n_1 < s_1 + s_2 + 1 \quad n_2 < s_1 + s_2 + 1 & &
\end{array}
\tag{1.1}
$$

Discontinuities described by II', III' and IV' are not of independent types and can be obtained by interchanging the subscripts 1 and 2 in II, III and IV.

For the discontinuities of the type I, II, II', III', IV' and V, the quantities to the left of the discontinuity and its velocity, can be assigned arbitrarily. For the types III, IV and VI, they should be connected by $s_1 + s_2 + 1 - n_2$ relations. Relations at the discontinuity yield unique values for the quantities to the right of the shock wave in case of the types I, II, III, III', IV and VI, and values with the accuracy of up to $n_2 - (s_1 + s_2 + 1)$ arbitrary parameters, for the types II', IV' and V.

Type I discontinuities correspond to the gas dynamic and MHD shock waves, and to detonation, while combustion process in gas dynamics and MHD corresponds to type VI.

In the case of ionizing shock waves with the normal magnetic field component given, the medium before the shock wave is described by nine quantities, ρ, T, $\mathbf{v}$, $\mathbf{H}_\tau$ and $\mathbf{E}_\tau$ and that behind the wave – by seven quantities, since E_τ can be expressed in terms of $\mathbf{v}$ and $\mathbf{H}$. The number $s_1 + s_2 + 1$ of the boundary conditions at the ionizing shock wave varies for the different types of waves [3–5]. Slow supersonic ionizing wave (seven relations) corresponds to the type II. Intermediate supersonic and slow subsonic waves (eight relations) correspond to the type IV. Fast supersonic and intermediate subsonic waves (nine relations) correspond to the type III.

2. Structure of the discontinuities and additional boundary conditions. We shall investigate the structure of discontinuities separating different media and obtain the required number of the boundary conditions connecting the values of the quantities at $x = -\infty$ and $x = \infty$, from the conditions of existence of that structure. The method adopted in our study will, basically, resemble that used in [11] for a single medium.

We shall assume that each medium, separated by the surface of discontinuity $x = 0$, is described by

$$A_{ij}^{(\alpha)} \frac{\partial u_j^{(\alpha)}}{\partial t} + B_{ij}^{(\alpha)} \frac{\partial u_j^{(\alpha)}}{\partial x} + C_i^{(\alpha)} + D_{ij}^{(\alpha)} \frac{\partial^2 u_j^{(\alpha)}}{\partial x^2} = 0 \quad (i, j = 1, 2, \ldots, N_\alpha) \tag{2.1}$$

$$\alpha = 1 \text{ when } x < 0, \quad \alpha = 2 \text{ when } x > 0$$

Here $A_{ij}^{(\alpha)}$, $B_{ij}^{(\alpha)}$, $C_i^{(\alpha)}$ and $D_{ij}^{(\alpha)}$ are functions of $u_k^{(\alpha)}$ $\alpha = 1, 2$. Some of the $u_k^{(1)}$ may coincide with certain $u_k^{(2)}$. In the particular case dealing with the structure of a discontinuity in a single medium, all $u_k^{(1)}$ coincide with $u_k^{(2)}$ and the systems given by (2.1) for $\alpha = 1$ and $\alpha = 2$, become identical.

We shall assume that the steady state solutions of (2.1) are continuous (the requirement that (2.1) has no characteristics stationary with respect to the discontinuity, will suffice). In addition, we shall assume that all spatially periodic small perturbation waves described by (2.1) and propagating across a homogeneous medium background decay (we shall call such systems dissipative [11]). These conditions will be fulfilled, if the matrix D_{ij} has a sufficient number of elements; thus, in the magnetohydrodynamic case it is sufficient that the heat conductivity as well as the volume and magnetic viscosity are all different from zero.

Equations of a perfect medium corresponding to (2.1) govern the large scale perturbations and are obtained by equating to zero the terms in (2.1) containing the lowest order derivatives

$$C_i^{(\alpha)} (u_k^{(\alpha)}) = 0 \tag{2.2}$$

$$A_{ij}^{(\alpha)} \frac{\partial u_j^{(\alpha)}}{\partial t} + B_{ij}^{(\alpha)} \frac{\partial u_j^{(\alpha)}}{\partial x} = 0 \tag{2.3}$$

Let us consider the case when the number of independent equations given by (2.2) is less than N_α. In this case we can write some of $u_k^{(\alpha)}$ in terms of the remaining ones and insert them into (2.3). We shall assume that the perfect system obtained in this manner is hyperbolic, denote its order by n_α and the number of its characteristics emanating from the discontinuity, by s_α.

Let us investigate the continuous solutions of (2.1) in each of the regions $x < 0$ and $x > 0$, satisfying some boundary conditions connecting $u_i^{(1)}$ and $u_i^{(2)}$ on the plane $x = 0$.

Solutions tending to constant values of $u_{k\infty}^{(\alpha)}$ as $x \to (-1)^{\alpha}\infty$, will represent the structure of the discontinuity. Obviously, $u_{k\infty}^{(\alpha)}$ should satisfy (2.2). Boundary conditions at the media interface may express the continuity or the change in some quantities, reflecting the change in the properties of the medium on the passage across the surface of the discontinuity.

Number of the boundary conditions depends on the properties of (1.1) and shall be obtained below. Let us linearize the system (1.1) in $u_{\infty}^{(\alpha)}$ and consider a solution of the form $\exp i(kx - \omega t)$. Equation (1.1) yields the following algebraic connection between ω and k

$$D^{(\alpha)}(\omega, k) = 0 \tag{2.4}$$

and from it we can obtain a set of values $k_1(\omega), k_2(\omega), \ldots, k_{M_\alpha}(\omega)$ (where M_α denotes the order of the system (2.1)), for every ω. From the previous assumption that spatially periodic small perturbations decay we can infer that when $\operatorname{Im}\omega > 0$, then the roots k_i lie on the real axis of the complex plane k. Functions $k_i(\omega)$ have no poles in the upper semiplane ω and are, consequently, continuous, since it is only in the neighborhood of a pole that real values of k corresponding to ω with $\operatorname{Im}\omega > 0$ can be found. Roots $k_i(\omega)$ lying in the upper semiplane of k correspond, when $\operatorname{Im}\omega > 0$, to the waves propagating to the right, and the roots lying in the lower semiplane – to the waves propagating to the left. Thus, when $\operatorname{Im}\omega > 0$, the waves emanating from the surface of the discontinuity decay with the distance, while the waves incoming at this surface, increase with increasing x. Let us denote the number of waves emanating from the discontinuity in the left and right direction by p_1 and p_2 and those incoming at the discontinuity – by q_1 and q_2.

Conditions at the boundary separating two media should define the amplitudes of the emanating waves and the motion of the discontinuity, therefore we require $p_1 + p_2 + 1$ boundary conditions. This represents the condition of evolutionarity for dissipative media. Analogous conclusions on the number of boundary conditions required at a stationary boundary for an arbitrary system of equations, was proved in [12]. When both media are identical, $p_1 + p_2 = p_1 + q_1 = p_2 + q_2$ equations describe the conditions of continuity of u_j and of the first derivatives with respect to x of those u_m whose second derivatives appear in (2.1). An extra boundary condition must prescribe the position of the surface on which these conditions are given.

In investigating the steady state solutions, due attention must be given to solutions of the type $\exp i(kx - \omega t)$ at the limit, when $\omega \to 0$. If $\omega \to 0$ from the upper semiplane, then no roots $k_i(\omega)$ may intersect the real axis of k or tend to some finite value different from zero. This follows from the assumption that all small periodic perturbations decay with time. Some of $k_i(\omega)$ will tend to zero as $\omega \to 0$. Perturbations corresponding to these roots will, obviously, be described by the perfect system (2.2) and (2.3). therefore the number of roots tending to zero will be equal to n_α.

These of the n_α roots $k_j(\omega)$ which correspond to the waves of the perfect system propagating to the right, will tend to zero as $\omega \to 0$ ($\operatorname{Im}\omega > 0$) from the upper semiplane of the complex plane k, while those corresponding to the waves propagating to the left, will tend to zero from the lower semiplane.

Computing the number of roots remaining in the upper and lower semiplanes of the complex plane k when $\omega = 0$ we find that $p_\alpha - s_\alpha$ waves decay on moving away from the discontinuity, n_α waves are independent of x, and $q_\alpha - n_\alpha + s_\alpha$ waves increase on moving away from the discontinuity. Let us denote some of the derivatives $\partial u_m^{(\alpha)}/\partial x$ by $w_m^{(\alpha)}$, $m = n_\alpha + 1, n_\alpha + 2, \ldots M_\alpha$ in such a manner, that only the first order derivatives in x are left in the system (2.1). Integral curves in the space $V^{(\alpha)}$ of varaibles $u_i^{(\alpha)}$ and $w_m^{(\alpha)}$ which do not tend to infinity with $x \to \infty$, tend to singular points lying on some surfaces $\Sigma^{(\alpha)}$ given by Eqs.(2.2) together with the equation $w_m = 0$.

Dimension of the surface $\Sigma^{(\alpha)}$ is equal to n_α, and this corresponds to the fact that the linearized system has n_α solutions independent of x, when $\omega = 0$. Each point of the surface $\Sigma^{(\alpha)}$ represents a singularity of system (2.1) with $p_\alpha + q_\alpha - n_\alpha$ characteristic directions corresponding to those values of $k_i(0)$ which are different from zero. As we said before, $p_\alpha - s_\alpha$ of these characteristic directions are occupied by the integral curves entering the singulatity point as $x \to (-1)^\alpha \infty$, and the remaining ones – by the integral curves emerging from the singularity point.

Set of integral curves entering any of the singularity points as $x \to (-1)^\alpha \infty$, is characterized by $p_\alpha - s_\alpha + n_\alpha$ arbitrary constants, of which n_α constants define the position of the singularity point on the surface $\Sigma^{(\alpha)}$, while $p_\alpha - s_\alpha$ constants describe the manner of approach of an integral curve to this singularity point. We can choose $u_{k\infty}^{(\alpha)}$ defining the state of the perfect medium in front and behind the jump, as the quantities describing the positions of the singularity points and denote the constants describing the manner of approach of the integral curve to a singularity point by $C_a^{(\alpha)}$, $a = 1, 2, \ldots p_\alpha - s_\alpha$.

Segments of the integral curves in the spaces $V^{(1)}$ and $V^{(2)}$ which enter the singularity points as $x \to (-1)^\alpha \infty$, and satisfy $p_1 + p_2 + 1$ boundary conditions at the boundary separating two media when $x = 0$, represent the structure of the shock wave. In general, any two singularity points, one of which lies on $\Sigma^{(1)}$ and the other on $\Sigma^{(2)}$, can be connected by a solution describing the structure of the shock wave. The coordinates $u_{k\infty}^{(1)}$ and $u_{k\infty}^{(2)}$ of such singularity points must however satisfy certain relations in order for such a solution to exist. Let us find the number of the required relations.

Conditions at the boundary separating two media can be written as

$$F_b\,(u_{k\infty}^{(\alpha)},\ C_a^{(\alpha)}) = 0 \tag{2.5}$$

$$(b = 1, 2, \ldots, p_1 + p_2 + 1;\ a = 1, 2, \ldots, p_\alpha - s_\alpha,\ \alpha = 1, 2)$$

which, on eliminating $C_a^{(\alpha)}$ yield $s_1 + s_2 + 1$ equations connecting $u_{k\infty}^{(1)}$ and $u_{k\infty}^{(2)}$

$$f_r\,(u_{k\infty}^{(1)},\ u_{k\infty}^{(2)}) = 0 \qquad (r = 1, 2, \ldots, s_1 + s_2 + 1) \tag{2.6}$$

Thus we see that the conditions of evolutionarity are fulfilled. The form of the equations (2.6) is in general defined by the properties of the dissipative system (2.1). Nevertheless, as we said in Sect. 1, in many cases the relations (2.6), or at least some of them, are known and independent of the properties of the dissipative system.

Laws of conservation of mass, impulse and energy are examples of such relations, as well as the relations expressing the continuity of the tangential components of the electric field and normal component of the magnetic field. We recall, that in Sect. 1 we have classified the above relations as fundamental, and the remaining relations given by (2.6) and depending generally on the properties of the dissipative system, as complementary.

Elimination of the variables $C_a^{(\alpha)}$ from (2.5) so as to obtain (2.6) is possible, provided that at least one nontrival solution of (2.5) exists and, that the determinant of at least one highest order minor of the matrix $G = \| \partial F_b / \partial C_a^{(\alpha)} \|$ is not zero. If the first condition is not fulfilled, then no solution exists representing the structure of the discontinuity. Existence of the structure should be specially confirmed for each particular case. If the matrix G is degenerate, then the values of the coefficients $C_a^{(\alpha)}$ are no longer unique, and the number of equations in (2. 5) will exceed $s_1 + s_2 + 1$. This obviously corresponds to nonevolutionary discontinuities. Solution describing the structure of such discontinuities, if it exists, will not be unique. The degeneracy of matrix G, or for that matter any other degeneracy, can be considered an exception, although, if the points for which $s_1 + s_2 + 1$ is less than the number of fundamental relations are chosen as the initial and final singularity points, then the degeneracy of G is predetermined. Cases of nonevolutionary waves have been encountered in MHD.

In some cases it may transpire that on one side of the discontinuity $p_\nu - s_\alpha = 0$ or that by (2. 5) all $C_\alpha^{(\alpha)}$ on this side are equal to zero. In this case the continuous variation of quantities appearing in the solution representing the structure of the discontinuity will be confined to one side of the interface only, while all quantities will be constant on the other side. When the above conditions hold on both sides of the interface, then regions in which all values are constant will also appear on both sides of this interface. If some $u_k^{(1)}$ coincide with $u_k^{(2)}$, and conditions at the interface stipulate their continuity, or the continuity of their derivatives, then in the latter case the perfect variables will be continuous at the discontinuity. Such ionization and recombination surfaces parallel to the magnetic field, were dealt with in [2].

It can easily be confirmed that the conclusion concerning the total number of the boundary conditions satisfied the perfect variables $u_{k\infty}^{(1)}$ and $u_{k\infty}^{(2)}$ at the discontinuity, remains valid in the case when the shock wave structure contains a layer defined by a system of equations which differ from those valid for the regions situated to the left and right of this layer. Existence of this type of shock waves in a gas in the presence of a magnetic field was shown in [13], and it was found that the conductivity of the gas was equal to zero in front and behind the wave, and different from zero within a certain layer contained within the structure of such a shock wave.

We note that the structure of the discontinuities considered above was regarded as a steady state solution of the dissipative equations. Discontinuities however exist which do not possess a steady state structure, such as, e.g., the tangential velocity jump in a perfect fluid, or a rotational discontinuity in MHD. Velocity of propagation of such discontinuities coincides with one of the characteristic velocities; conclusions of this paper do not however apply to such discontinuities.

3. Recombination discontinuities in a gas in the presence of an arbitrarily orientated magnetic field. To illustrate our previous arguments, we shall consider the recombination discontinuities in a gas in the presence of a magnetic field.

We shall assume that the magnetic field is arbitrarily inclined to the surface of the discontinuity. Recombination waves parallel to the magnetic field were considered earlier in [2].

We shall also assume that the conductivity of the gas is a function of the density and temperature $\sigma = \sigma(\rho, T)$ and, that $\sigma > 0$ in some (ρ, T)-space of variables, whose boundary is given by the equation $F(\rho, T) = 0$, and is identically zero outside this space.

Basic boundary conditions follow from the continuity of the mass flux, the impulse (three equations), the energy, the continuity of the tangential electric field component (two equations) and of the normal magnetic field component.

When obtaining the additional boundary conditions in their relevant form, we assume that all dissipative coefficients are appreciably smaller than the value of the magnetic viscosity.

Equations describing the structure of the recombination waves coincide with those for the ionizing waves. The only difference is that in the case of recombination waves we must find the integral curve originating at one of the singularity points situated within the area $\sigma > 0$ and terminating at one of the singularity points within the area $\sigma = 0$, while in the case of inonizing waves it is the other way round. Using the data of the singularity points and on the character of the integral curves given in [5] which deals with ionizing shock waves, we can easily construct solutions representing the structure of the recombination waves. Methods of constructing such solutions are well presented in [1–5 and 13], consequently we shall just quote the final results.

Let us denote the normal gas velocity component in front and behind the discontinuity by u_1 and u_2, respectively; gasdynamic velocity of sound, density and temperature behind the discontinuity by a_2, ρ_2 and T_2 and the velocities of propagation of the fast Alfvenian and the slow perturbations in front of the discontinuity by a_+, a_A and a_- respectively.

Next we shall write the conditions governing the gas velocities u_1 and u_2 for four possible types of recombination waves together with the additional boundary conditions obtained for the case when the magnetic viscosity is much higher than that of the remaining dissipative coefficients. When the relations between the dissipative coefficients are changed, then the additional boundary conditions may assume a different form, but their number will remain the same

1. $$u_1 > a_+, \qquad u_2 > a_2, \qquad F(\rho_2, T_2) = 0 \tag{3.1}$$

2. $$a_A < u_1 < a_+, \qquad u_2 < a_2, \qquad F(\rho_2, T_2) = 0 \tag{3.2}$$

3. $$a_+ < u_1 < a_+, \qquad u_2 > a_2, \qquad F(\rho_2, T_2) = 0, \qquad H_y = f(H_z^2) \tag{3.3}$$

4. $$a_- < u_1 < a_A, \qquad u_2 < a_2, \qquad F(\rho_2, T_2) = 0, \quad \Delta H_z \equiv H_{z1} - H_{z2} = 0 \tag{3.4}$$

Last relation of (3.3) represents the equation of the integral curves emerging, with increasing x, from the singularity point corresponding to the state in front of the wave. Form of the function f can be found either by numerical integration, or by constructing the solution in form of a series. Relations (3.3) and (3.4) ar written in a coordinate system, in which $H_{z1} = 0$. For the waves of the type given by (3.4), the inequality $u_1 < a_1$, where a_1 is the gasdynamic velocity of sound in front of the wave, also holds.

If the dependence of the gas conductivity on ρ and T is such that the conductivity is different from zero in front of a certain gasdynamic shock wave and equal to zero behind it, then additional two types of recombination waves are possible,

5. $$u_1 > a_+, \qquad u_2 < a_2 \tag{3.5}$$

There are no additional relations in this case

6. $$a_- < u_1 < a_A, \qquad u_2 < a_2, \qquad \Delta H_y \equiv H_{y1} - H_{y2} = 0, \qquad \Delta H_z = 0 \tag{3.6}$$

With waves of this type we have the inequality $u_1 > a_1$.

In all these waves, hydrodynamic parameters and the magnetic field undergo a change (with the exception of the waves described by (3.6) where the magnetic field is constant). In addition, we have the recombination waves in which the hydrodynamic parameters and the magnetic field are continuous. Their spatial position and motion are defined by the temperature and density fields in a continuous flow, by the relation $F(\rho, T) = 0$ which, together with the equations $\Delta H_y = 0$ and $\Delta H_z = 0$ can be regarded as three additional relations for these waves. The velocity of gas $u = u_1 = u_2$ in such waves, should satisfy one of the following inequalities

$$u < a_-, \qquad a < u < a_A \tag{3.7}$$

where $a = a_1 = a_2$ denotes the hydrodynamic velocity of sound.

BIBLIOGRAPHY

1. Kulikovskii, A. G. and Liubimov, G. A., On the magnetohydrodynamic, gas-ionizing shock waves. Dokl. Akad. Nauk SSSR, Vol. 129, №1, 1959.
2. Butler, D. S., One dimensional flow in ionizing gas. J. Fluid Mech. Vol. 23, Pt. 1, 1965.
3. Taussig, R. T., Comparison of oblique, normal and transverse ionizing shock waves. Phys. of Fluids, Vol. 10, №6, 1967.
4. Cowley, M. D., Gas-ionizing shocks in magnetic field. J. Plasma Phys., Vol. 1, №1, 1967.
5. Barmin, A. A. and Kulikovskii, A. G., Gas-ionizing shocks in electromagnetic field. Dokl. Akad. Nauk SSSR, Vol. 178, №1, 1968.
6. Akhiezer, A. I., Liubarskii, G. Ia. and Polovin, R. V., Stability of shock waves in MHD. JETP, Vol. 35, №3, 1958.
7. Landau, L. D. and Lifshits, E. M., Mechanics of Continuous Media. M., Gostekhizdat, 1953.
8. Barmin, A. A., Surfaces of discontinuity with emission (absorption) of energy in MHD. PMM Vol. 26, №5, 1962.
9. Polovin, R. V. and Demutskii, V. P., On the magnetohydrodynamic combustion. JETP, Vol. 40, №6, 1961.
10. Soubmayer, Sur les chocs dans un milieu magnétodynamique réactif avec application un problème du piston. Thes. Doct. sci. math. Fac. sci. Univ. Paris Rapp. CEA, №3137, 1967.
11. Liubarskii, G. Ia., On the structure of shock waves. PMM Vol. 25, №6, 1961.
12. Hersh, B., Boundary conditions for equations of evolution. Arch. Rat. Mech. and Anal., Vo . 16, №4, 1964.
13. Zhilin, Iu. L., A particular case of the interaction of a strong, stationary shock wave in a gas, with an electromagnetic field. Izv. Akad. Nauk SSSR, OTN, Mechanika i mashinostroenie, №2, 1962.

Translated by L. K.